Econ 802

Second Midterm Exam

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All questions have equal weight. If something is unclear, please ask. In all questions you should assume that prices and income are strictly positive.

1. The direct utility function is $u(x_1, x_2) = ax_1 + bx_2$ where $a > 0$ and $b > 0$. There is a non-negativity constraint $(x_1, x_2) \geq 0$.

(a) Derive the indirect utility function $v(p_1, p_2, m)$. Be sure the value of the function is specified for every $(p, m) > 0$. Explain your reasoning using a graph.

(b) Derive the expenditure function $e(p_1, p_2, u)$. Be sure the value of the function is specified for every $(p, u) > 0$. Explain your reasoning using a graph.

(c) Initially prices and income are given by $(q, m) > 0$. Now the price vector changes to $p \neq q$. Assume the consumer's optimal bundle is <u>different</u> at the prices q and p. What level of income m' is required at the prices p in order to make the consumer exactly as well off as she was with (q, m)? Explain your answer using a graph.

2. Assume the direct utility function $u(x)$ is differentiable and strictly quasi-concave, where $x = (x_1 \, . \, . \, x_n) \geq 0$. Also assume local non-satiation.

(a) Suppose x^* maximizes $u(x)$ subject to $px \leq m$. Write $u^* = u(x^*)$. Prove that x^* minimizes px subject to $u(x) \geq u^*$.

(b) For an arbitrary consumption bundle $x^* > 0$, how would you find prices $p^* > 0$ and income $m^* > 0$ such that the consumer wants x^* at (p^*, m^*)? Derive these prices mathematically and explain your reasoning.

(c) For an arbitrary consumption bundle $x^* > 0$, prove that $u(x^*) = \min v(p, 1)$ subject to $px^* = 1$ where the minimization is with respect to p. Use a graph to provide some intuition for the two-good case.

3. An individual consumer has the Marshallian demand functions $x_1(p, m) = m/2p_1$ and $x_2(p, m) = m/2p_2$.

(a) Consider a discrete price change $\Delta p_2 > 0$. There is no change in p_1 or m. Using a graph, split the total effect $\Delta x_1 = 0$ into a substitution effect and an income effect. Describe your procedure in words.

(b) For the demand functions given above, write out the Slutsky equation using 2 x 2 matrices and solve for the substitution matrix $\partial h/\partial p$. Does the substitution matrix have the properties we would usually expect? Explain.

(c) Suppose there are many consumers $i = 1 \,..\, n$. Consumer i has income m_i where different consumers could have different incomes. All consumers face the same prices $p > 0$. Let $X^1(p, m_1 \,..\, m_n)$ and $X^2(p, m_1 \,..\, m_n)$ be the aggregate demands for goods one and two. Do the aggregate demands depend on the distribution of income? What would probably happen if you repeated the same computations as in part (b) using the aggregate demand functions? What is probably true about the indirect utility functions for the individual consumers? Explain. Note: you do not need to repeat part (b) or solve for the indirect utility functions. Just discuss using words and a small amount of mathematical notation.

4. Assume local non-satiation. In period $t = 1$, we see that a consumer faces the price vector $p^1 = (3, 2)$ and chooses $x^1 = (6, 4)$. In period $t = 2$, we see that the same consumer faces the price vector $p^2 = (3, 5)$ and again chooses $x^2 = (6, 4)$.

(a) Is there any utility function that could have led to these observations? Explain using a graph.

(b) Taking the first two observations as given, construct a third numerical observation (p^3, x^3) that would be consistent with GARP (the Generalized Axiom of Revealed Preference). Explain using a graph.

(c) Taking the first two observations as given, construct a third numerical observation (p^3, x^3) that violates GARP. Explain using a graph.

5. Here are some miscellaneous questions.

(a) State the sufficient second order condition for utility maximization. Then use a graph and words to interpret this condition.

(b) Write the consumption bundle as (x, z). The prices of the x goods are given by p, the prices of the z goods are given by q, and the price ratios among the individual z goods never change. Explain how to create a price index t for the z goods, and show that the Marshallian demands for the x goods depend only upon p/t and m/t.

(c) An undergraduate student has heard that the elasticity of leisure with respect to the wage is not significantly different from zero. The student interprets this to mean that leisure is "almost" a Giffen good. What would you say in response?